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Numerical simulations on cracking process in ceramic specimens under thermal shock by using a non-local fracture model

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Abstract

In this paper, we present a finite element approach to fracture modeling for brittle or quasi-brittle materials and its application to thermal shock crack simulations. The proposed fracture model is constructed on the basis of the conventional maximal principal stress criterion for uniform tensile loads and the Griffith-Irwin criterion for crack propagation prediction. Consequently, the proposed fracture criterion can be used to predict both the crack initiation and the crack growth. By using the proposed model, we carried out detailed numerical simulations on cracking process of ceramic materials subjected to thermal shock loading. The random aspect of strength and toughness of the material was considered by generating stochastically their spatial distribution. The comparison with the experimental results shows that the periodic and hierarchical structure of the crack pattern is faithfully reproduced by the numerical simulations.

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Keywords: Fracture, non-local model, multi-cracking, thermal shock, crack pattern

1. Introduction

Ceramic materials are widely used in various industries due to their excellent thermal, electrical and mechanical properties. However, their inherent brittleness make them particularly vulnerable to thermal shock failure. In general, crack formation is considered as the major reason of failure in thermo-structural engineering. Understanding the mechanisms of cracking process in ceramics under thermal loads has been one of the most importance tasks in the research of this field. In this paper, we will develop a stochastic non-local fracture model in

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order to simulate the multi-cracking process appeared in ceramic specimens under thermal shock loading. The ideas described in Li et al. (2012,2013) were essentially followed. The random strength and toughness distributions in the material were established according to the Weibull model (1939) obtained from the macroscopic strength tests and the microscopic analysis. Using the established non-local damage model, we carried out detailed numerical simulations on cracking process of the ceramic specimens under thermal shock loading. The proposed model in the present work allows a highly realistic description of the multi-cracking process. Comparison with the experiments shows that the predicted crack paths and the stochastic characteristics of the crack patterns are faithfully reproduced.

2. A non-local fracture criterion

In this Section, we describe brievely the non-local fracture model for brittle and quasi-brittle materials introduced in Li et al. (2012, 2013) . The start point is the commonly used maximum stress fracture criterion $\sigma_1 \geq \sigma_c$ for brittle material. According to this criterion, the element will be instantaneously broken when the first principal stress σ_1 reaches the material strength σ_c . However, this criterion is not suitable to predict fractures due to crack growth. In order to overcome this shortcoming, Li et al. (2012, 2013) made use of the so-called non-local approach (Pijaudier-Cabot and Bazan, 1987) in order to establish a new fracture criterion. The non-local tress field can be calculated as follows:

$$\widetilde{\sigma}_{ij}(\mathbf{x}) = \int_V \varphi(\mathbf{x}-\mathbf{y})\sigma_{ij}(\mathbf{y})d\mathbf{y} \tag{1}$$

where V is the non-local interaction volume, $\varphi(\mathbf{x}-\mathbf{y})$ is a space weighting function. In 2D calculations, we define V as a circle of radius R and centered at the source point **x.** For simplicity, a cone-shaped function is chosen as the weighting function φ:

$$\varphi(r) = \begin{cases} 0 & r > R \\ \dfrac{3}{\pi R^2}\left(1-\dfrac{r}{R}\right) & r \leq R \end{cases} \tag{2}$$

where $r = \|\mathbf{x}-\mathbf{y}\|$ is the distance between the points **x** and **y**. Thus, a new fracture criterion based on the maximum stress criterion writes simply $\widetilde{\sigma}_1 \geq \sigma_c$, where $\widetilde{\sigma}_1$ is the non-local first principal stress. In order to determine the parameter R, we enforce the non-local fracture criterion to be exact in two special cases, as follows:

1. It must be exact for fracture prediction under uniform tensile loading.
2. It must be exact for propagation prediction of a mode-I crack.

The first condition can always be satisfied. The second condition can be satisfied by considering the near-tip stress field of a mode-I crack and by connecting the non-local fracture criterion to the Griffith-Irwin crack growth criterion. After simple algebraic deduction, we can determine R and the position where the non-local principal stress is maximal characterized by the distance r_0 between this point and the crack tip. Numerical calculations show that rather an accurate estimation of these values can be obtained by using the following simple formulas:

$$R = \frac{1.05}{\pi}\left(\frac{K_{Ic}}{\sigma_c}\right)^2 \qquad r_0 = 0.4R \tag{3}$$

As the proposed non-local fracture criterion is exact for crack onset prediction under uniform tensile loading and for the propagation prediction of a mode-I crack, the fracture prediction under other types of stress concentrations can just be considered as a natural interpolation between the two former special cases. Consequently, the proposed fracture criterion can be applied to predict the crack onset as well as the crack growth with quite a high accuracy.

This non-local fracture model was implemented into a finite element code. The crack propagation evaluation is just a sequence of linear analyses on the cracked structure. We first calculate the local and non-local stress fields and find the position where the non-local first principal stress is maximal; then we create a new crack in the direction perpendicular to the non-local first principal stress. The length of the newly created crack can be estimated by using the energy balance analysis. The elements crossed by the crack are then removed from the finite element model. The successive crack growth will form the final crack pattern.

3. Apllication to thermal shock problem

3.1 Experiments

Quenching tests on ceramic specimens were carried out in order to observe the cracking patterns due to the thermal shock (Jiang et al, 2012). 99% Al_2O_3 bulk specimens were cut into 50mm×10mm×1mm thin plates. They were tightly tied together with metallic wires and heated to a temperature T_0 ranged from 300°C to 600°C. After that, the heated specimens were dropped into a water bath of T_∞=20°C by free fall. 2D cracks can be observed on the surface of the specimens (Figure 4). The crack pattern presents some typical characteristics. First, a great number of cracks appear and form a complicated, periodic and hierarchical crack pattern. Second, the number of cracks increases whereas the crack spacing decreases as the initial temperature increases. We also observe a tendency towards equal spacing between cracks. In the following, we will attempt to reproduce these properties of crack patterns by direct numerical simulations.

3.2 Finite element model and material properties

The mechanical and thermal characteristics are assumed to be independent of the temperature variation. Most of these parameters are taken as constants. From available data, their values are listed in Table 1.

Table 1. Mechanical and thermal parameters used in the simulations

E(the Young module, MPa)	ν (Poisson's ratio)	ρ (mass density, kg/m^3)
370000	0.3	3980
k (thermal conductivity W/(m.K))	c (specific heat, J/(kg.K))	α (thermal expansion coefficient)
31	880	7.5×10^{-6}

However, some important fracture parameters such as the ultimate stress σ_c and the toughness G_c cannot be considered as constant for ceramic materials. In fact, the values of these parameters depend on the shape and size of the tested specimen. Moreover, the measured values exhibit important scattering behaviour. In order to represent these effects in fracture modelling, we introduce the Weibull fracture model (1939) into the finite element calculations. We assume that the material strength follows the Weibull distribution, the probability of a material of size V with strength less than σ is

$$P(\sigma)=1-\exp\left(-\frac{V}{V_0}\left(\frac{\sigma}{\sigma_0}\right)^m\right) \qquad (4)$$

where σ_0 and m are scale and shape parameters, respectively, V_0 is a reference volume. The establishment of the Weibull probability can be obtained by carrying out the macroscopic strength tests such as the 3-point bending test. Let $\mathbf{X}(\mathbf{x},\omega)$ be a stationary Gaussian random field of the material strength, where $\mathbf{x}$ denotes the Cartesian coordinate vector of any point in the domain and ω a random sample. It can be estimated from the corresponding Weibull field $\mathbf{Y}(\mathbf{x},\omega)$ and vice versa. The correlation function of 2D Gaussian random field $\mathbf{X}(\mathbf{x},\omega)$ is chosen as

$$C(\mathbf{x}_1-\mathbf{x}_2)=s^2\exp\left(-\frac{\pi|\mathbf{x}_1-\mathbf{x}_2|^2}{l_c^0}\right) \qquad (5)$$

where s is the variance and l_c is the correlation length which is determined by a microscopic structure analysis. Since l_c is numerically close to the correlation length of $\mathbf{Y}(\mathbf{x},\omega)$, it can be used to indicate the characteristic length of a heterogeneous medium, e.g., the average aggregate size in ceramics. The Weibull strength model and the parameters describing the random strength field were determined by means of macroscopic 3-point bending fracture tests and the microscopic structure analysis. The obtained parameters are listed in Table 2

Table 2. Strength and toughness parameters

$\bar{\sigma}$ (mean strength, MPa)	s (strength variance MPa)	l_c (strength correlation length, mm)
288	29	0.1
$\bar{G}$ (mean toughness, MPa.m)	g (toughness variance MPa)	L_c (toughness correlation length, mm)
3	0.3	0.1

By symmetry, only a quarter of the specimen is modelled in finite element simulations. Three node triangular

elements were used with uniform size over whole the specimen. The size of the elements is determined such as the smallest value of R is superior to two times of the element size. From this condition, the size of the elements is fixed as 0.031mm. The random fields of the strength and toughness were generated by using the turning band method. Figure 1 shows one of the random strength fields used in numerical simulations.

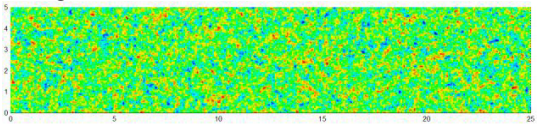

Fig. 1. Random strength field generated by using the turning band method

4. Numerical results and discussions

In numerical simulations, the temperature field during the quenching test was first calculated by the finite element method for each step of times. The thermal stress field was then found in the specimen. After that the proposed non-local fracture model was applied to assess the crack evolution. In the following, the principal numerical results are presented.

4.1 Crack initiation at the specimen surfac

In Figure 2, we show the positions of the first crack onsets and the non-local first principal stress distribution in the specimen. When the non-local criterion is fulfilled at a point at a critical time (about 10^{-3} second after the quenching), a crack is created according to the proposed criterion. As a consequence, the stress relaxation takes place at the vicinity of this point. Then the algorithm will find another position where the non-local principal stress is maximal and create a crack if the fracture criterion is fulfilled. This procedure will repeat until the temperature gradient is no longer capable to produce sufficient stress concentration to onset new cracks. Then a time increment will provide a new temperature map that may probably lead to new cracking process.

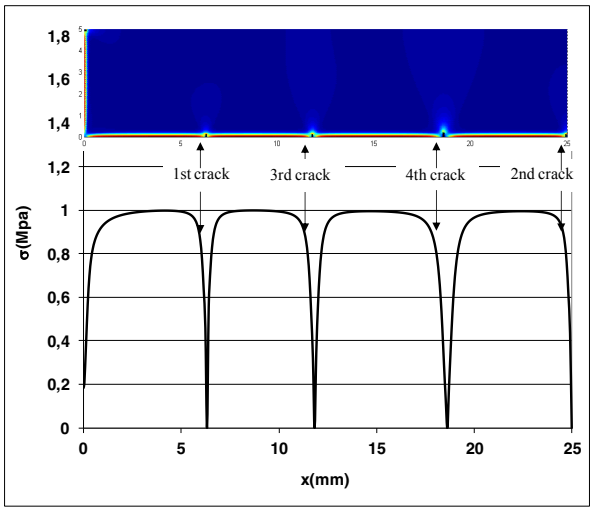

Fig. 2. First crack onsets and stress redistribution on the specimen border

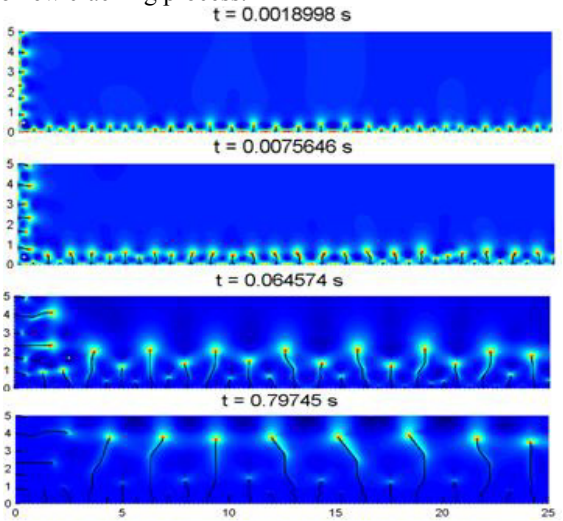

Fig. 3. Crack patterns at different times for T_0 = 500°C.

In Figure 3, the crack patterns at different times during the quenching test are illustrated. At the beginning, all the cracks appear within a very short time with a nearly equal spacing, then they propagate all together and rapidly. Then the propagation speed decreases gradually until some of them stop growing. Only a reduced number of cracks continue to propagate such that the crack spacing increases until 2 or 3 times larger. In the following steps, the crack growth may deviate and attempt to form equal crack spacing.

4.2 Direct comparison with the experiments

Figure 4 illustrates the final crack patterns of the numerical simulations for different initial quenching temperatures. Globally speaking, the simulation results reproduce all the essential characteristics of the real crack patterns: Multiple cracks are obtained for different initial quenching temperatures; The cracks initiate on the contour and propagate into the interior of the specimens; Periodical and hierarchical crack structure is correctly reproduced; We also observe a tendency towards equal spacing between cracks during all the cracking process as in real specimens.

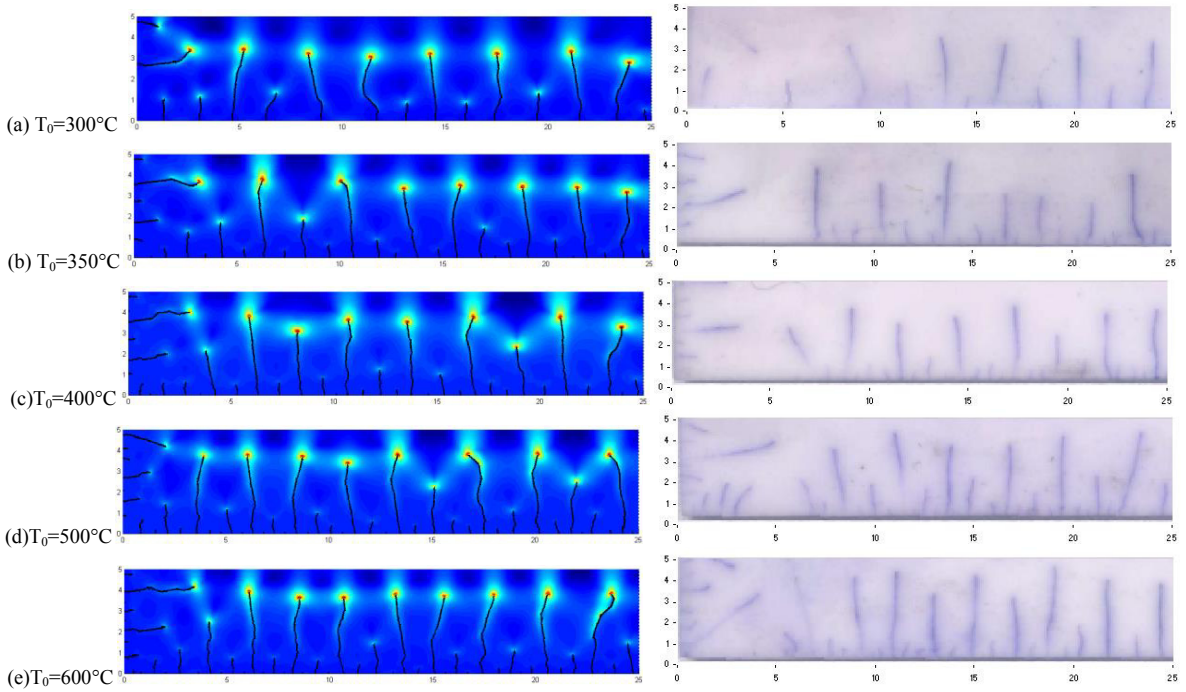

Fig. 4. Comparison between the crack patterns in numerical models and in real specimens.

4.3 Statistical results

By using the generated strength and toughness random fields, we carried out 10 numerical simulations for each quenching temperature. In this manner, the random effects revealed in the experiments can naturally be taken into account. As the strength of ceramic materials exhibits important scattering behaviour and depends on the shape and size of the specimen, an "accurate" strength value does not exist for all types of stress states. From this point of view, the stochastic numerical simulation is the only reasonable way to evaluate the fracture of this class of materials.

In Figure 5, we compare the relationships between the average crack spacing and the crack length issued from the numerical and experimental statistics. The average crack spacing s has been determined by counting the intersection points of cracks with a straight line at the depth p. These values were normalized by the semi-width of the specimens to obtained the corresponding dimensionless values $\bar{s} = s/L_2$ and $\bar{p} = p/L_2$. The experimental results are obtained from statistics over 5 specimens. From Figure 5, we can observe that the numerical results agree very well with the experiments, especially in the case of high quenching temperatures. Comparing with the early numerical results (Li et al. 2013), the stochastic simulations improve considerably the quality of the quantitative results.

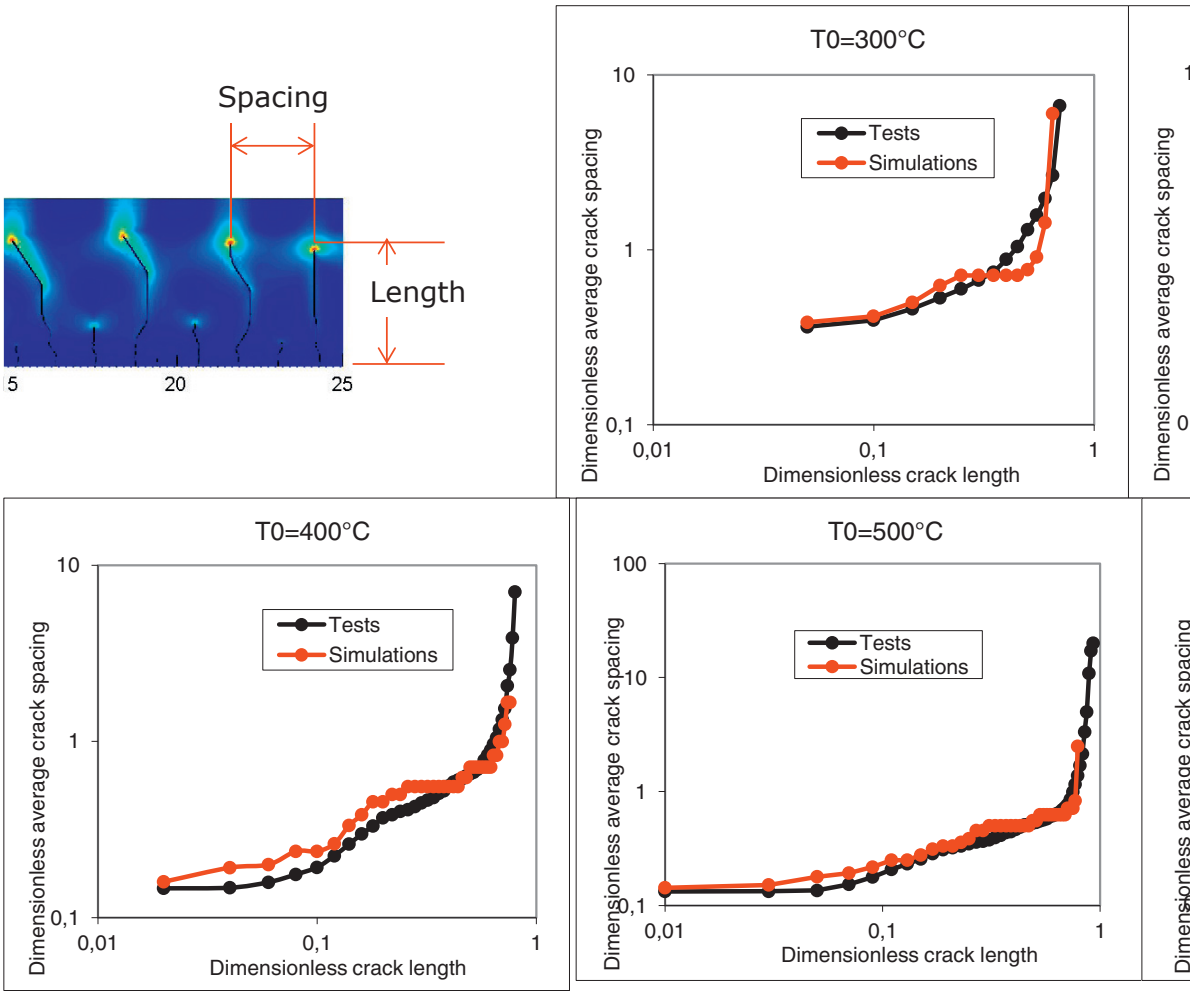

Fig. 5. Normalized crack spacing $\bar{s} = s/L_2$ vs normalized crack length $\bar{p} = p/L_2$.

5. Concluding remarks

In this work, a non-local failure model was used to predict crack initiation and crack growth in a 2D structure made of brittle materials. It was implemented into a finite element code. The random strength and toughness fields were generated according to the Weibull model established from macroscopic tests and microscopic structure analysis. This non-local fracture model was successfully applied to simulating the crack evolution in ceramic materials subjected to thermal shock. The numerical simulation reproduced faithfully the multi-cracking process and the crack patterns in ceramic specimens subjected to quenching tests. The periodical and hierarchical characteristics of the crack patterns were accurately predicted. The stochastic parameters describing the crack patterns such as the average crack spacing and the crack lengths were correctly estimated from the numerical results. The numerical simulations allow a direct observation on crack initiation and growth in the specimens, which is quite a difficult task in experimental studies.

Acknowledgements

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